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The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

A computational approach is developed for optimal path planning for constrained nonlinear dynamical systems. In the approach developed here, the continuous-time optimal control problem is transcribed to a finite-dimensional nonlinear programming problem. In this research, we develop novel methods for discretization based on Legendre-Gauss and Legendre-Gauss-Radau quadrature points. Using this approach, the finite-dimensional approximation is kept low-dimensional, potentially enabling near real time or real time solutions. The approach is

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a. REPORT	b. ABSTRACT	c. THIS PAGE			Anil Rao
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Report Title

A Computational Approach for Near-Optimal Path Planning and Guidance for Systems with Nonholonomic Contraints

ABSTRACT

A computational approach is developed for optimal path planning for constrained nonlinear dynamical systems. In the approach developed here, the continuous-time optimal control problem is transcribed to a finite-dimensional nonlinear programming problem. In this research, we develop novel methods for discretization based on Legendre-Gauss and Legendre-Gauss-Radau quadrature points. Using this approach, the finite-dimensional approximation is kept low-dimensional, potentially enabling near real time or real time solutions. The approach is demonstrated on sample problems and is found to be a highly accurate and computationally efficient way to discretize constrained nonlinear optimal control problems.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Garg, D., Patterson, M. A., Hager, W. W, Rao, A. V., Benson, D. A., and Huntington, G. T., "A Unified Framework for the Numerical Solution of Optimal Control Problems Using Pseudospectral Methods," Provisionally Accepted to Automatica, December 2009.

Rao, A. V., Benson, D. A., Darby, C. L., Patterson, M. A., Francolin, C., and Huntington, G. T., "Algorithm 902: GPOPS, A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using The Gauss Pseudospectral Method," ACM Transactions on Mathematical Software, Vol. 37, No. 2, April-June 2010, to appear. See http://toms.acm.org/Upcoming.html for more information.

Garg, D., Patterson, M. A., Darby, C. L. Francolin, F., Huntington, G. T., Hager, W. W, and Rao, A. V., "Direct Trajectory Optimization and Costate Estimation of Finite-Horizon and Infinite-Horizon Optimal Control Problems Using a Radau Pseudospectral Method," Computational Optimization and Applications, Published Online at SpringerLink, 6 October 2009. See http://www.springerlink.com/content/n851q6n343p9k60k.

Number of Papers published in peer-reviewed journals: 3.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

Garg, D., Patterson, M. A., Darby, C. L. Francolin, F., Huntington, G. T., Hager, W. W, and Rao, A. V., "Direct Trajectory Optimization and Costate Estimation of Finite-Horizon and Infinite-Horizon Optimal Control Problems Using a Radau Pseudospectral Method," SIAM Annual Meeting, Denver, Colorado, July 2009.

Number of Presentations: 1.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Darby, C. L. and Rao, A. V., "An Initial Examination of Using Pseudospectral Methods for Time-Scale and Differential Geometric Analysis of Nonlinear Optimal Control Problems," 2008 Astrodynamics Speclaist Conference, AIAA Paper 2008-6449, Honolulu, Hawaii, August 18-21, 2008.

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 1

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

Number of Manuscripts: 0.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED
Divya Garg	0.23
FTE Equivalent:	**0.23**
Total Number:	**1**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Anil V. Rao	0.24	No
William W. Hager	0.53	No
FTE Equivalent:	**0.77**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME	
Divya Garg	
Total Number:	**1**

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

A Computational Approach for Near-Optimal Path Planning and Guidance for Systems with Nonholonomic Constraints

Final Report

Grant Number 55173-CI

Anil V. Rao and William W. Hager
University of Florida
Gainesville, FL 32611-6250

Objective

The objective of this research is to develop a computational framework for simplified path planning and near-optimal guidance law development for problems with nonholonomic constraints using a synthesis of orthogonal collocation methods and computational differential geometry. A secondary objective of this research is to develop a theoretical foundation for orthogonal collocation methods to enhance the ability to employ differential geometric techniques in developing simplified path plans. This research is highly relevant to future Army needs in autonomous ground vehicle mission planning, guidance, and control.

Approach

The technical approach developed in the proposed research combines advanced numerical methods for solving optimal control problems with computational differential geometry. In particular, orthogonal collocation methods, which have been shown to have high accuracy in both the primal and dual solutions, will be used to determine extremal trajectories to constrained nonlinear optimal control problems. Then, exploiting the accuracy of the dual solution, the Hamiltonian dynamics corresponding to the optimal control problem can be analyzed using approaches from nonlinear dynamical system theory. Specifically, finite-time Lyapunov exponents can be used to develop insight into the regions where the various time-scales in the problem are active while finite-time Lyapunov vectors can be used to extract the differential geometric structure in the problem. The synthesized framework provides a novel way to access the structure of the Hamiltonian dynamics, but without ever having to solve the (often intractable) Hamiltonian boundary-value problem that arises from the calculus of variations. Extraction of time-scale and differential geometric structure in the Hamiltonian system then provides a novel opportunity to reduce the complexity of the model and arrive at simplified guidance laws.

Scientific Barriers

The analysis of Hamiltonian systems is an important part of understanding and characterizing the structure of optimal path planning problems. Moreover, the vast majority of complex optimal path planning problems are nonlinear, have no closed-form analytic solutions, and thus must be solved and analyzed computationally (i.e., using numerical methods). The scientific barriers in this research are (1) the ability to obtain accurate approximations to Hamiltonian phase space trajectories that arise in nonlinear optimal path planning problems; (2) the ability to analyze the optimal path plans in order to better characterize the underlying structure of the optimally controlled systems; and (3) a convergence theory for the numerical methods used to determine the optimal path plans.

Significance

The significance of the results of this research will be an advanced computational framework for developing autonomous path planning, guidance, and control algorithms in future-generation unmanned Army ground vehicles. In particular, the motion of unmanned wheeled ground vehicles is governed by highly nonlinear differential equations and these vehicles operate in highly constrained environments. Moreover, because of the requirement for autonomy, the demands of the on-board guidance system is significantly higher as compared to vehicles operated by humans. Because of the required sophistication of the modeling together with the required performance, it is difficult to conceive simplified path planning and guidance algorithms for autonomous ground vehicles using purely analytical methods. As a result, computational methods that enable extraction of simplified guidance laws are of great benefit to the Army. The proposed framework will be an enabling technology for simplified path planning and guidance for unmanned ground vehicles, leading to the use of these algorithms on board future Army vehicles.

Accomplishments

The following key aspects of the research were accomplished this past year:

- A new global orthogonal collocation method for solving nonlinear finite-horizon and infinite-horizon optimal control problems using collocation at Legendre-Gauss-Radau (LGR) points has been researched and developed. The results of this research are being presented at a mini-symposium on pseudospectral methods at the 2009 SIAM Annual meeting in Denver, Colorado and have been submitted for journal publication as given in Ref. [1].

Progress has also been made toward developing a unified framework for the solution of optimal control problems using orthogonal collocation methods and a journal article detailing the results of this research is currently being written. This research will provide a rigorous basis for the reliable solution of complex optimal control problems that arise in autonomous ground vehicle mission planning, guidance, and control.

- A preliminary study of time-scale and differential geometric analysis in vehicle dynamics applications. A conference paper on this research was published as given in Ref. [2]. Currently, this work is being transformed into a form suitable for journal publication. Finally, we are currently researching problems with nonholonomic constraints and plan to have at least one article on the topic later in the year.

- An open-source mathematical software for solving multiple-phase optimal control problems was developed. An article detailing the results of this research has been accepted for journal publication as given in Ref. [3].

Collaborations and Leveraged Funding

There are no collaborations of this research with other organizations or individuals. Leveraged funding for this research includes student fellowship and teaching assistantship funds provided by The University of Florida.

Conclusions

The progress this past year on this research has been excellent. We have developed a new method for solving optimal control problems, made significant progress characterizing the methods used in this research, applied the methodologies to problems of relevance to the U.S. Army, and developed a new software platform for wide use in the academic, government, and not-for-profit research communities.

Technology Transfer

The key technology transfer that has been provided by this research is the aforementioned open-source software tool GPOPS (see Ref. [3]) which is available for worldwide download at http://sourceforge.net/projects/gpops. Further information about GPOPS can be found at http://gpops.sourceforge.net.

References

[1] Garg, D., Patterson, M. A., Darby, C. L., Francolin, C., Huntington, G. T., Hager W. W., and Rao, A. V., "Direct Trajectory Optimization and Costate Estimation of Finite-Horizon and Infinite-Horizon Optimal Control Problems Using a Radau Pseudospectral Method," *Computational Optimization and Applications*, submitted for publication, March 2009.

[2] Darby, C. L. and Rao, A. V., "An Initial Examination of Using Pseudospectral Methods for Time-Scale and Differential Geometric Analysis of Nonlinear Optimal Control Problems," *2008 Astrodynamics Specialist Conference*, AIAA Paper 2008-6449, Honolulu, Hawaii, August 18-21, 2008.

[3] Rao, A. V., Benson, D. A., Darby, C. L., Patterson, M. A., Francolin, C., Sanders, I., and Huntington, G. T., ``Algorithm: GPOPS, A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using the Gauss Pseudospectral Method,'' *ACM Transactions on Mathematical Software*, accepted for publication, June 2009 (to appear).